BOROUGH GREEN
Primary School

Calculations Policy

Mathematics Mastery

At the centre of the Mastery approach to the teaching of Mathematics is the belief that **all children have the potential to succeed**. They should have access to the same curriculum content and, rather than being extended by new learning, **they should deepen their conceptual understanding by tackling challenging and varied problems**. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 Primary National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). It is essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary including the development of STEM sentences. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

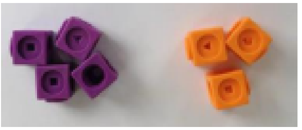
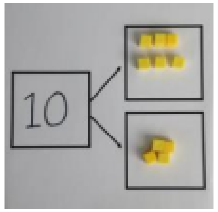
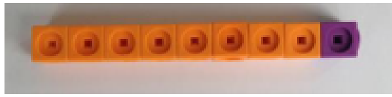
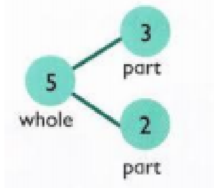
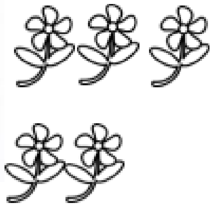
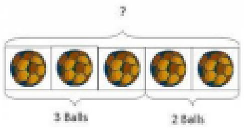
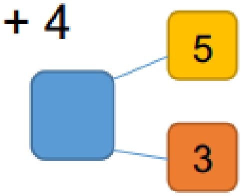
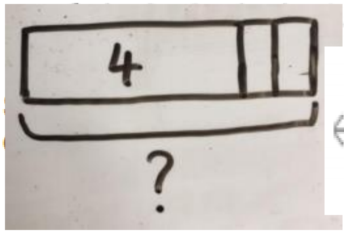
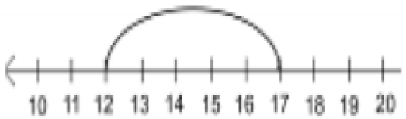
How to use the policy

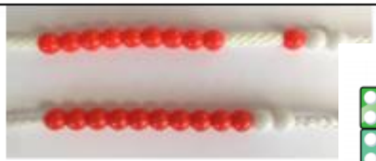
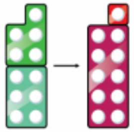
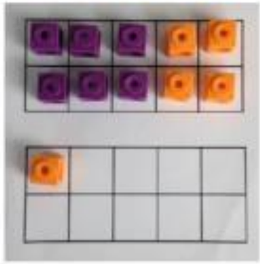
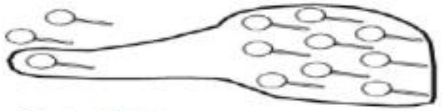
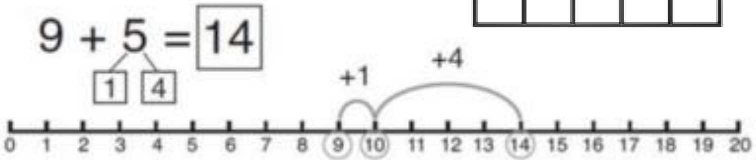
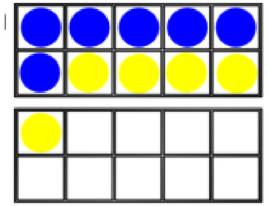
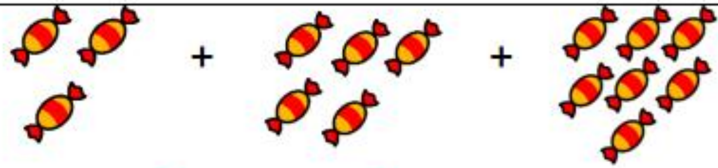
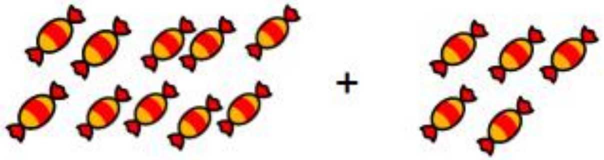
This mathematics policy is a guide for all staff at Borough Green Primary School and has been adapted from the work by NCETM and White Rose Maths Hub. It is set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement and the Ready To Progress Criteria to decide when consolidation of existing skills is required or pupils and classes are ready to move onto the next concept. However, **the focus must always remain on breadth and depth rather than accelerating through concepts.** Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems through conceptual and procedural variation. All teachers have access to the scheme of work from the White Rose Maths Hub including the mixed year group planning and are required to base their planning around their year group modules and not to move onto a higher year group's scheme of work. These modules use the Singapore Maths Methods and are affiliated to the workings of the 2014 Maths Programme of Study. Teachers have also been directed to the materials produced by the NCETM and the Maths Hub (mastery and greater depth).

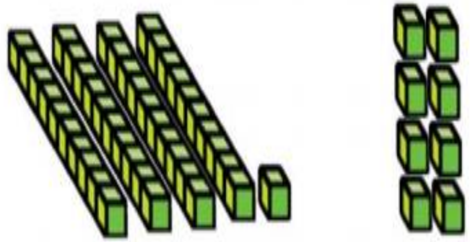
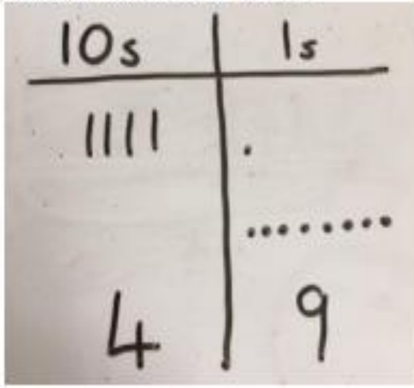
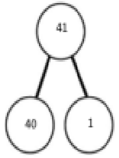
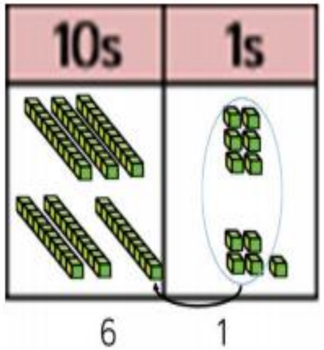
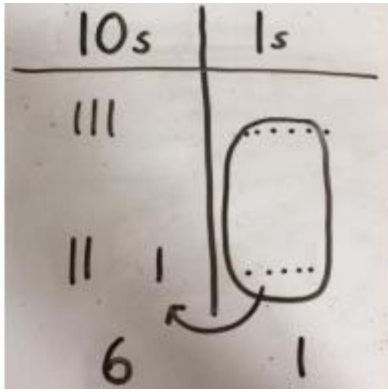
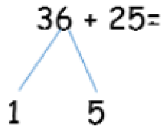
The policy supports the use of a variety of concrete manipulatives and visual representations. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract approach (Make it, Draw it, Write it) is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

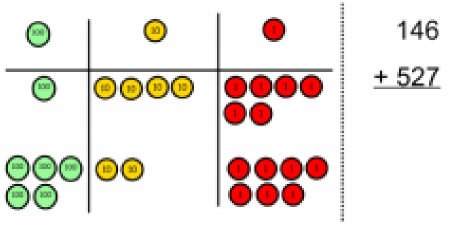
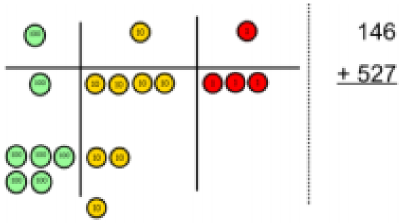
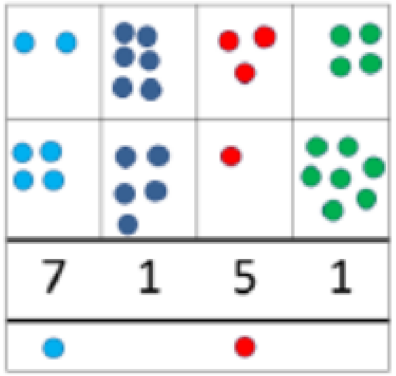
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

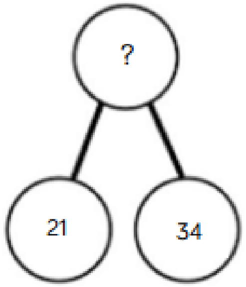
	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17 A box model which encourages the children to count on, rather than count all.  Start at the larger number on the number line and count on in ones or in one jump to find the answer. 	5 + 12 = 17 The abstract number line: What is 12 more than 5? What is the sum of 12 and 5? What is the total of 5 and 12? Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.	  6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10.  Children to draw the ten frame and counters/cubes. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	  Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.

Adding a one digit number to a two digit number	**TO + O using base 10.** Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens dot/crosses for ones.  	41 + 8  1 + 8 = 9 40 + 9 = 49 $\begin{array}{r} 41 \\ +\ \ 8 \\ \hline 49 \end{array}$
Adding a two digit number to a two digit number – no regrouping	**TO + TO using base 10.** Continue to develop understanding of partitioning and place value. 36 + 25  	Children to represent the base 10 in a place value chart.  	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: $\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$

Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. $\begin{array}{ccc} 20 & + & 5 \\ \underline{40} & \underline{+} & \underline{8} \\ 60 & + & 13 \end{array} = 73$ $\begin{array}{r} 536 \\ \underline{+\ 85} \\ \underline{621} \\ 1\,1\ \end{array}$ As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. $\begin{array}{r} 72.8 \\ \underline{+\ 54.6} \\ \underline{127.4} \\ 1\ 1\ \ \ \end{array}$ $\begin{array}{cccccc} £ & 2 & 3 & . & 5 & 9 \\ + \ £ & & 7 & . & 5 & 5 \\ \hline £ & 3 & 1 & . & 1 & 4 \\ \hline & 1 & 1 & & 1 & \end{array}$ $\begin{array}{cccccc} & 2 & 3 & . & 3 & 6 & 1 \\ & & 9 & . & 0 & 8 & 0 \\ & 5 & 9 & . & 7 & 7 & 0 \\ + & & 1 & . & 3 & 0 & 0 \\ \hline & 9 & 3 & . & 5 & 1 & 1 \\ \hline & 2 & 1 & & 2 & & \end{array}$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

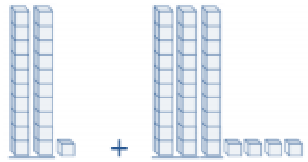
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\boxed{}$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

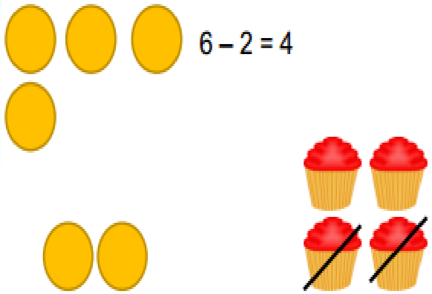
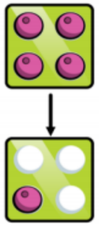
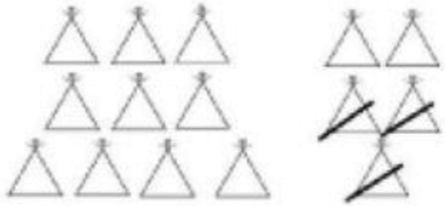
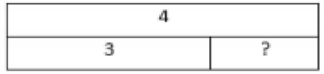
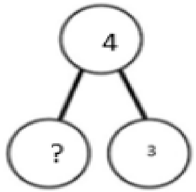
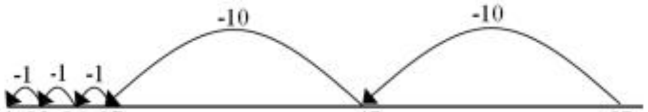


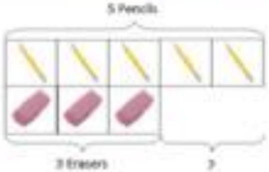
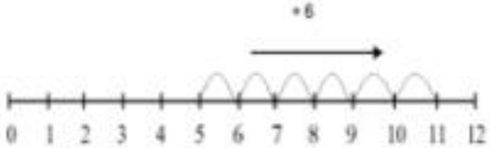
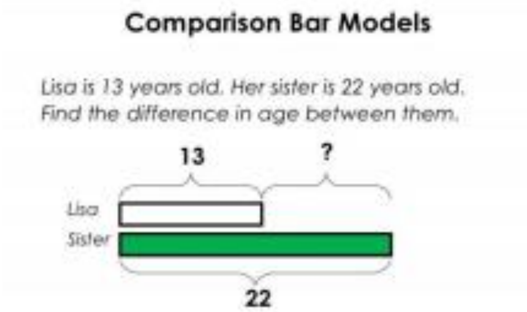
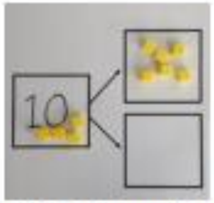
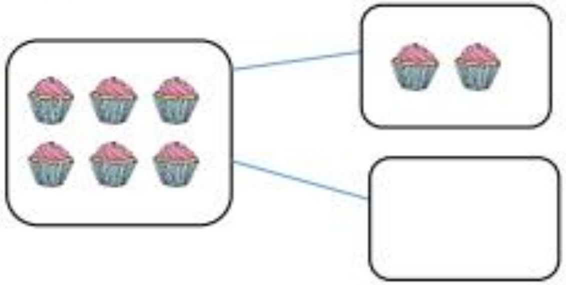
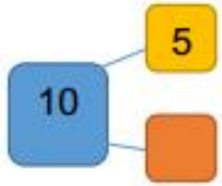
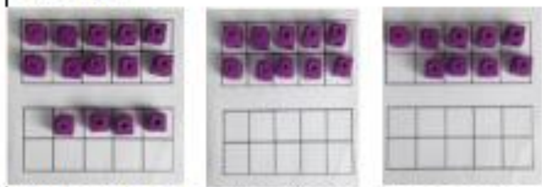
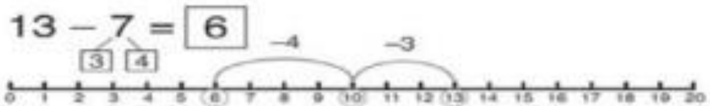
Missing digit problems:

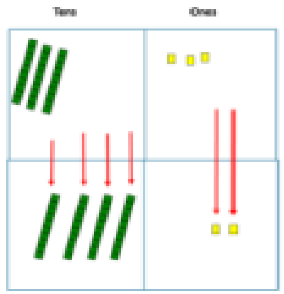
10s	1s
10 10	1
10 10 10	?
?	5

Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

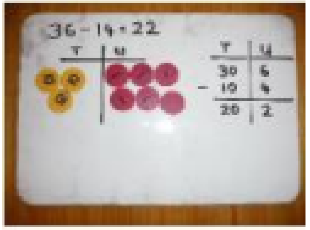
	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.   6 – 2 = 4	Cross out drawn objects to show what has been taken away.  15 – 3 = $\boxed{12}$	4- 3 = $\boxed{}$ = 4 – 3  4 / 3, ?  4 / ?, 3
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -1 -1 -1 -10 -10 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

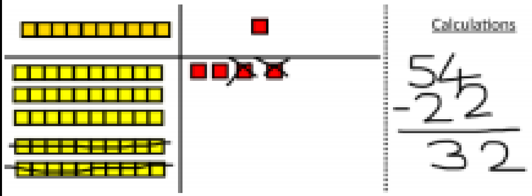
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  	Find the difference between 8 and 5. 8 - 5, the difference is $\boxed{}$ Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

<table>
<tr>
<td>Column method without regrouping</td>
<td>


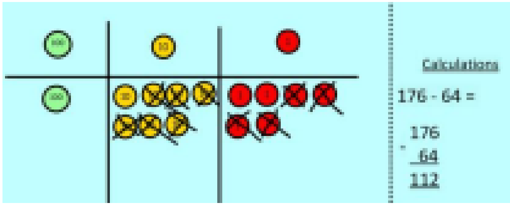
Use Base 10 to make the bigger number then take the smaller number away.

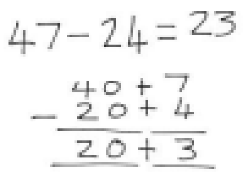
Show how you partition numbers to subtract. Again make the larger number first.



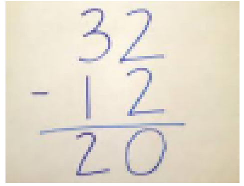
</td>
<td>


Draw the Base 10 or place value counters alongside the written calculation to help to show working.



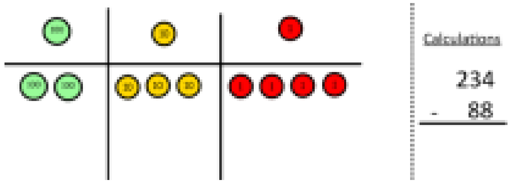
</td>
<td>


This will lead to a clear written column subtraction.

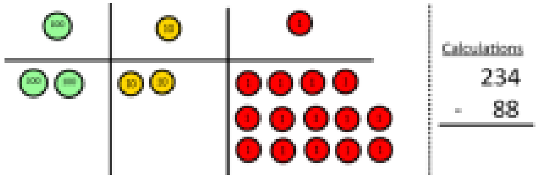


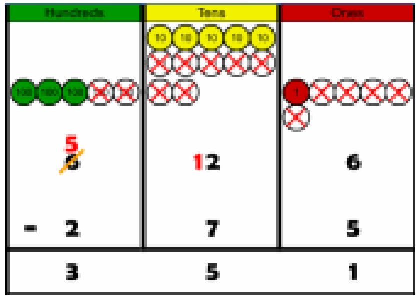
</td>
</tr>
<tr>
<td>Column method with regrouping</td>
<td>
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

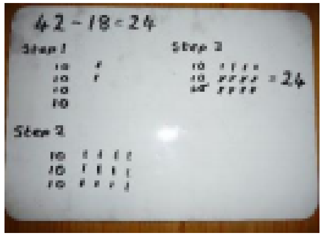


Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

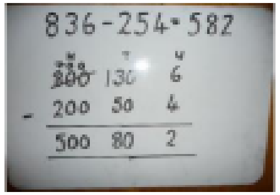


</td>
<td>


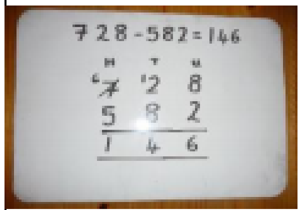
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



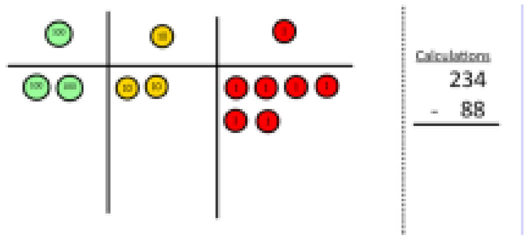
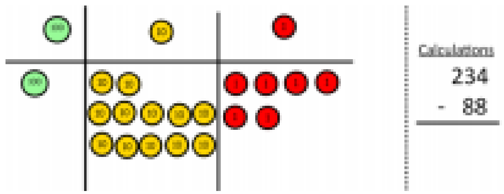
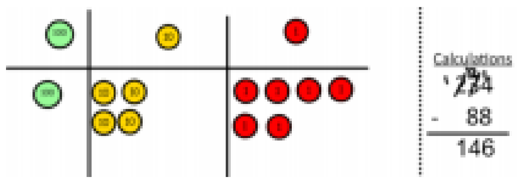
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.
</td>
<td>


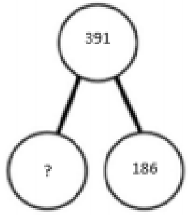
Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.
</td>
</tr>
</table>

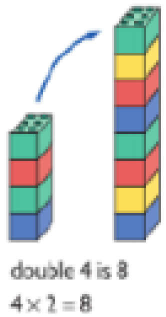
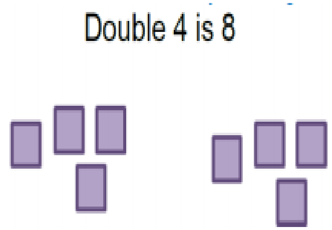
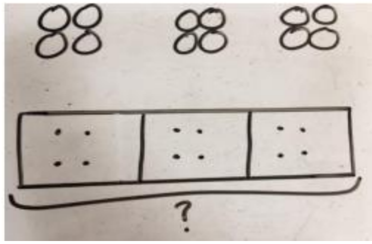
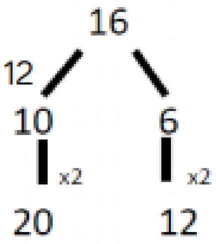
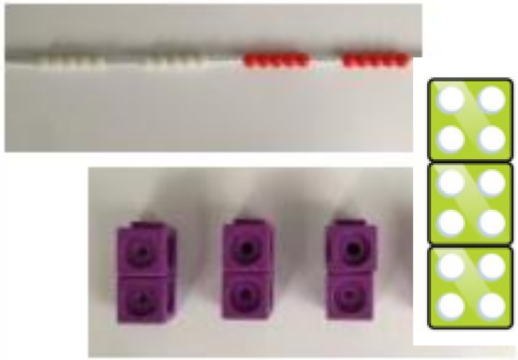
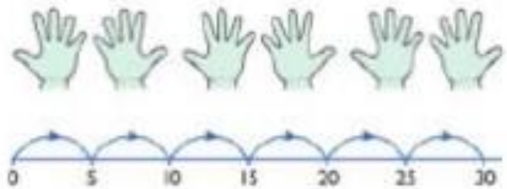
	Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. $\begin{array}{cccccc} & & {}^{5} & {}^{12} & & {}^{1} \\ & 2 & \cancel{6} & \cancel{3} & . & 0 \\ - & & 2 & 6 & . & 5 \\ \hline & 2 & 3 & 6 & . & 5 \end{array}$

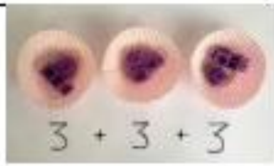
Conceptual variation; different ways to ask children to solve 391 - 186

 391 186 | ?	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	$\square$ = 391 – 186 391 -186 ____ What is 186 less than 391?	Missing digit calculations 3 9 $\square$ - $\square$ $\square$ 6 ______ $\square$ 0 5

Calculation policy: Multiplication

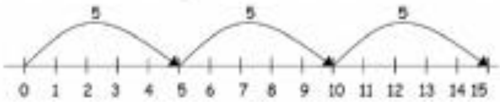
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  Children to represent the practical resources in a picture and use a bar model.  	$3 \times 4 = 12$ $4 + 4 + 4 = 12$  Partition a number and then double each part before recombining it back together.
Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

<table>
<tr>
<td>Repeated addition</td>
<td>

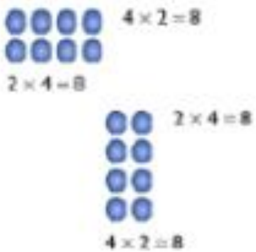


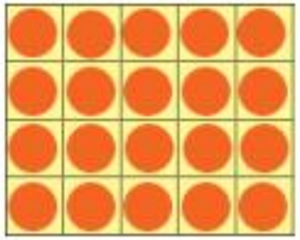
Use different objects to add equal groups.</td>
<td>



5 + 5 + 5 = 15</td>
<td>Write addition sentences to describe objects and pictures.

</td>
</tr>
<tr>
<td>Arrays- showing commutative multiplication</td>
<td>Create arrays using counters/ cubes to show multiplication sentences.

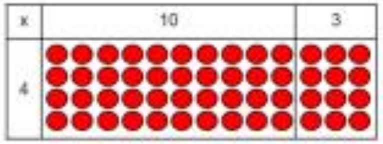
</td>
<td>Draw arrays in different rotations to find commutative multiplication sentences.



Link arrays to area of rectangles.</td>
<td>Use an array to write multiplication sentences and reinforce repeated addition.

5 + 5 + 5 = 15
3 + 3 + 3 + 3 + 3 = 15
5 x 3 = 15
3 x 5 = 15</td>
</tr>
</table>

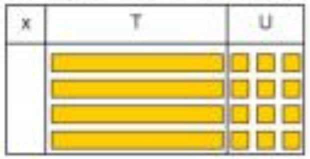
Grid Method

Show the link with arrays to first introduce the grid method.



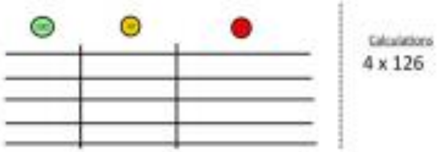
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

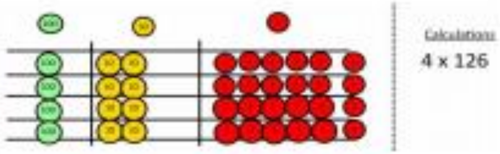


4 rows of 13

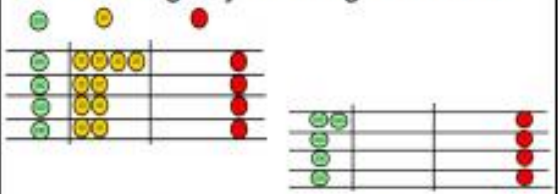
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



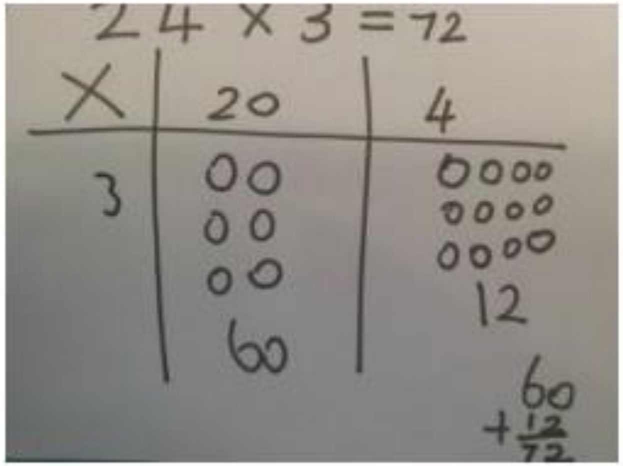
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	**30**	**5**
7	**210**	**35**

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	**8**
10	**100**	**80**
3	**30**	**24**

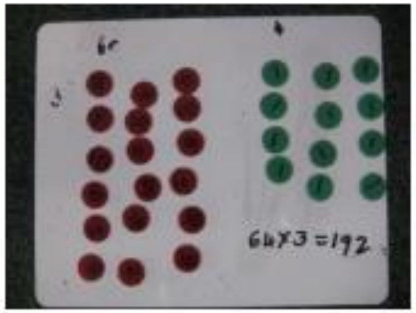
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

<table>
<tr>
<td>

Column multiplication

</td>
<td>

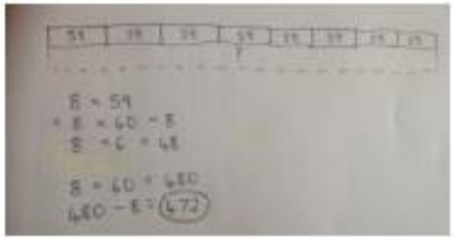
Children can continue to be supported by place value counters at the stage of multiplication.

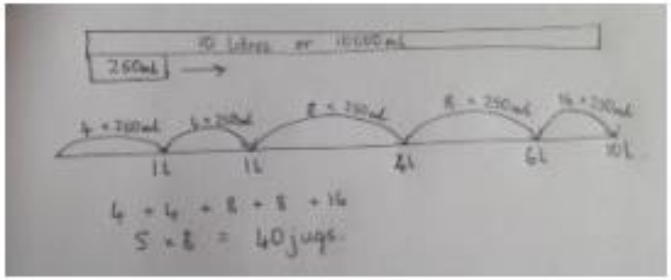


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

</td>
<td>

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





</td>
<td>

Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

```
   32
 x 24
    8   (4 x 2)
  120   (4 x 30)
   40   (20 x 2)
  600   (20 x 30)
  768
```

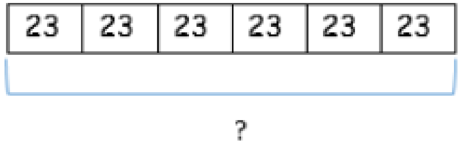
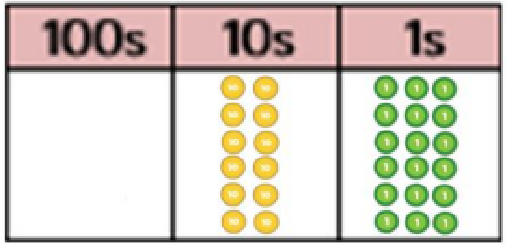
```
       7 4
   x   6 3
       1 2
     2 1 0
     2 4 0
 + 4 2 0 0
   4 6 6 2
```

This moves to the more compact method.

```
   2 3   1
   1 3 4 2
 x     1 8
 1 3 4 2 0
 1 0 7 3 6
 2 4 1 5 6
   1
```

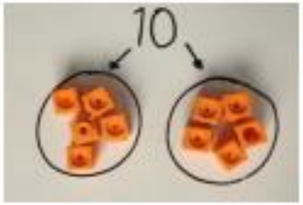
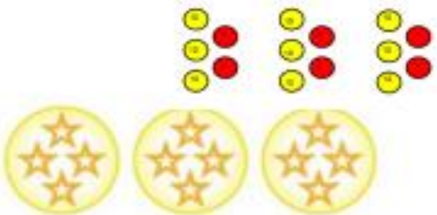
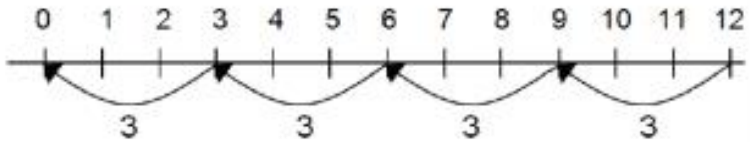
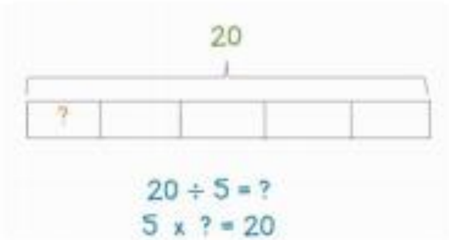
</td>
</tr>
</table>

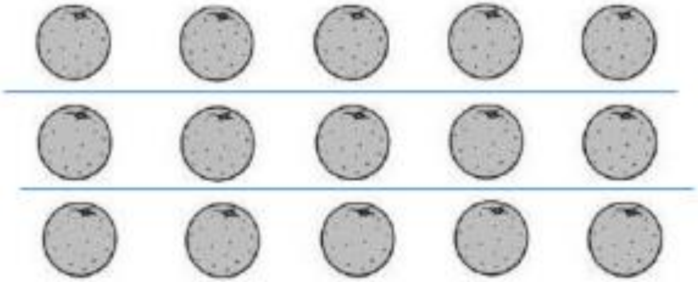
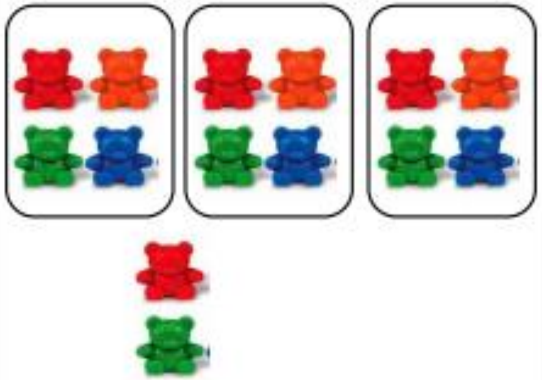
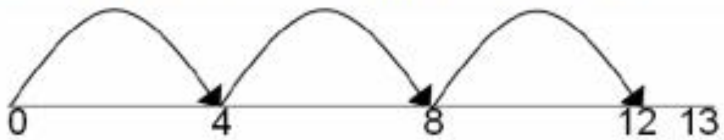
Conceptual variation; different ways to ask children to solve 6 × 23

	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that 6 x 23 = 138	Find the product of 6 and 23 $6 \times 23 =$ $\boxed{} = 6 \times 23$ $\begin{array}{r} 6 \\ \times\ \underline{23} \\ \underline{} \end{array}$ $\begin{array}{r} 23 \\ \times\ \underline{\ 6} \\ \underline{} \end{array}$	What is the calculation? What is the product? 

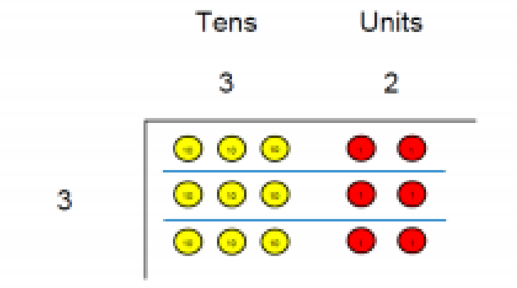
Calculation policy: Division

Key language: share, group, divide, divided by, half.

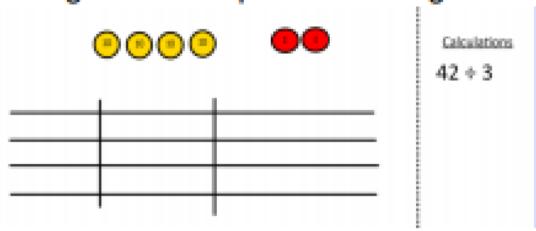
	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$ $6 \div 2 = 3$ | 3 | 3 | Children should also be encouraged to use their 2 times tables facts.
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   $96 \div 3 = 32$ 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 

Short division

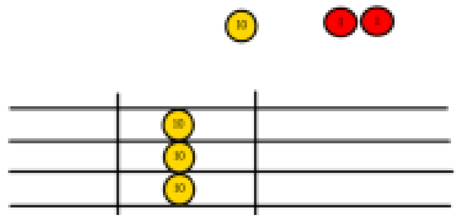


Use place value counters to divide using the bus stop method alongside

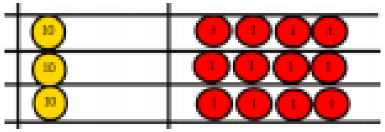


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

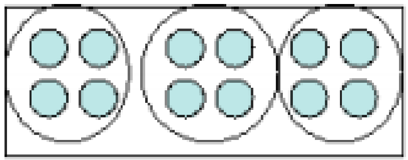


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$

$42 = 30 + 12$

$30 \div 3 = 10$

$12 \div 3 = 4$

$10 + 4 = 14$

Begin with divisions that divide equally with no remainder.

```
    2  1  8
   ---------
   |     3
 4 | 8  7  2
```

Move onto divisions with a remainder.

```
     8  6   r 2
   ---------
   |    3
 5 | 4  3  2
```

Finally move into decimal places to divide the total accurately.

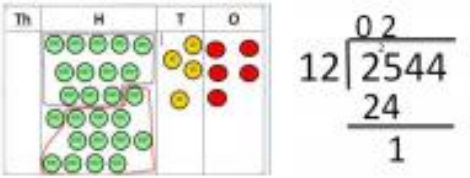
```
          1   4  .  6
       -----------------
       |     16     21
 3  5  | 5  1   1  .  0
```

Long division

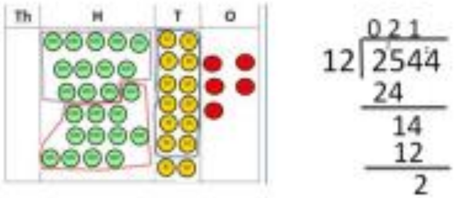
2544 ÷ 12
How many groups of 12 thousands do we have? None

Exchange 2 thousand for 20 hundreds.

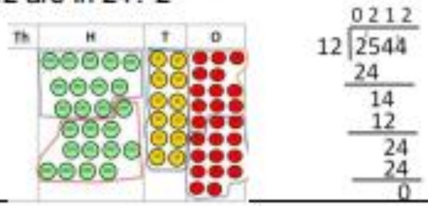
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2

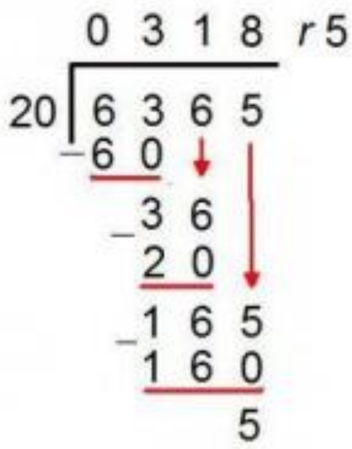


Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

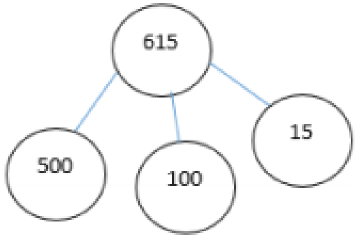


Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.



Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division? 	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = $\square$ = 615 ÷ 5	What is the calculation? What is the answer?